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Coincident (super)-Dp-branes of codimension one

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Abstract: We consider properties of a covariant worldvolume action for a system of N coincident Dp-branes in $D = (p+2)$ dimensional space-time (so called codimension one branes). In the case of N coincident D0-branes in $D = 2$ we then find a generalization of this action to a model which includes fermionic degrees of freedom and is invariant under target-space supersymmetry and worldline kappa-symmetry. We find that the type-IIA $D = 2$ superalgebra generating the supersymmetry transformations of the ND0-brane system acquires a non-trivial "central extension" due to a nonlinear contribution of U(N) adjoint scalar fields. Peculiarities of space-time symmetries of coincident Dp-branes are discussed.

Keywords: p-branes, D-branes, Space-Time Symmetries, Supersymmetry and Duality.

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Contents

1. Introduction

Systems of N coincident Dirichlet p-branes play an important role in String Theory. In particular, their low energy dynamics gives rise to effective supersymmetric non-abelian (Born-Infeld-type) field theories with an internal gauge group U(N). This is why intensive research have been undertaken to get a detailed information about the structure and the dynamics of these multibrane configurations. Such an information can be obtained if one knows the worldvolume actions which describes the low energy behaviour of the Dp-branes.

Actions for bosonic systems of N coincident Dp-branes have been constructed in [1, 2, 3] using T-duality tools and comparison with results known from Matrix Theory.

These actions are written in a static gauge in which (p+1) coordinates of the target-space are identified with the worldvolume coordinates σ^a $(a = 0, 1, \ldots, p)$, and target-space indices $M, N, \ldots$ split into the worldvolume indices $a, b, \ldots$ and the indices $i, j \ldots = 1, \ldots, D - p - 1$ of target-space coordinates transverse to the D-branes. Thus the space-time symmetry of the background is explicitly broken down to a direct product of the worldvolume symmetry times an internal symmetry acting on the indices $i, j, \ldots$

The worldvolume action for N coincident Dp-branes in a D-dimensional bosonic supergravity background has the following form

$$S = -T_p \int d^{p+1}\sigma \,\mathrm{Tr}\, e^{-\phi} \sqrt{-\det\left(P\left[E_{ab} + E_{ai}(Q^{-1}-\delta)^{ij} E_{jb}\right] + F_{ab}\right) \det(Q^i{}_j)} + S_{WZ}\,, \tag{1.1}$$

where T_p is the brane tension and ϕ is the dilaton field. $F_{ab} = \partial_a A_b - \partial_b A_a + \frac{i}{2\pi l_s^2}[A_a, A_b]$ is the field strength of a worldvolume U(N) gauge field $A_a(\sigma)$. In addition, the action (1.1) depends on $D-p-1$ scalar fields $\Phi^i(\sigma)$ and on their covariant derivatives $D_a\Phi^i = \partial_a \Phi^i + \frac{i}{2\pi l_s^2}[A_a, \Phi^i]$ taking values in the space of the adjoint representation of U(N) (i.e. they are $N \times N$ hermitian matrices).[1]

P denotes the pull back onto the worldvolume of the matrix

$$E_{MN} = G_{MN} + B_{MN} \tag{1.2}$$

composed of the target-space metric G_{MN} and an NS-NS field B_{MN}. The explicit form of the pullback P of E_{MN} (1.2) is

$$P[E]_{ab} = E_{ab} + E_{ai}\, D_b\Phi^i + D_a\Phi^i\, E_{ib} + D_a\Phi^i D_b\Phi^j\, E_{ij}\,. \tag{1.3}$$

The matrix $Q^i{}_j$ in (1.1) has the form

$$Q^i{}_j \equiv \delta^i{}_j + \frac{i}{2\pi l_s^2}\,[\Phi^i, \Phi^k]\, E_{kj}\,. \tag{1.4}$$

The diagonal elements of the $N \times N$ matrix Φ^i are associated with transverse space coordinates of the N Dp-branes when they are separated.

The term S_{WZ} in (1.1) is a Wess-Zumino term which describes the coupling of the N Dp-brane system to lower as well as to higher-rank RR fields, the coupling to the latter is possible because of the non-commutativity of the adjoint scalar fields Φ^i [3, 2].

Finally, it should be noted that all the background fields in the non-abelian action (1.1) are assumed to be the same functionals of the adjoint scalars Φ^i as they are in the abelian theory of a single Dp-brane [4, 5]. Further details on the structure of the action (1.1) the reader may find in [3].

Leaving apart a topical problem of the (symmetrized) trace of the non-abelian Dirac-Born-Infeld theory [6], one can also be unsatisfied that the action (1.1) is in the static gauge and, hence, is not worldvolume reparametrization invariant. This action is also not invariant under target space diffeomorphisms (except for the case of the space-filling branes, i.e. pure non-abelian DBI theories, where there is no scalar fields), though it is invariant under the gauge transformations of the background RR fields, as has been recently proved in [7].

[1] In our convention A_a and Φ^i have the dimension of length in the string scale l_s, in contrast to [3] where their dimension is l_s^{-1}.

The lack of worldvolume and target space diffeomorphism invariance may be a reason why there is a problem with supersymmetrizing the multiple brane actions (in target space and on the worldvolume (making them κ-symmetric)) and thus with introducing worldvolume fermions into the construction. By now supersymmetrization and fermion couplings have been considered only for D0-branes in the context of Matrix theory [2] and for non-abelian DBI theories [8, 9, 10]. In [9] a possibility of constructing a supersymmetric space-filling N Dp-brane action with a non-abelian κ-symmetry has been considered up to F^2 terms in the variation, but then the conclusion has been made [10] that such a construction would fail beyond the quadratic approximation.

In this note we will address ourselves to studying the above mentioned (super)symmetry problems of N coincident Dp-branes.

A natural way would be to start from the sum of N reparametrization invariant and κ-symmetric actions for N separated Dp-branes and analyze how their symmetries get modified because of the contribution of the non-abelian fields when the N D-branes coincide,[2] but this program seems to be rather complicated and ambitious, as has been also discussed in [9].

So to approach somehow these problems we suggest to look at the system of N coincident Dp-branes from a somewhat different point of view. Namely, let us consider them as a qualitatively new *single* brane configuration which is created when N Dp-branes stack up together. We shall call it the N Dp-brane. The trace part

$$x^i(\sigma) = \frac{1}{N}\mathrm{Tr}\Phi^i \tag{1.5}$$

of the U(N) adjoint scalars Φ^i (which is the center of mass of the N branes) is then naturally identified with the transverse coordinates of this single brane object in the target space-time. Together with the worldvolume coordinates σ^a they may be regarded as N Dp-brane coordinates $x^M(\sigma) = (\sigma^a, x^i(\sigma))$ $(m = 0, 1, \ldots, D)$ in a D-dimensional target space in the static gauge. We can now leave the static gauge by making the theory worldvolume diffeomorphism invariant introducing $p+1$ coordinates $x^a(\sigma)$:

$$x^M(\sigma) = (x^a(\sigma), x^i(\sigma))\,, \quad (M = 0, 1, \ldots, D)\,. \tag{1.6}$$

If we admit this point of view then the U(N) vector fields $A_a(\sigma)$ and the 'traceless' scalars

$$\varphi^i(\sigma) \equiv \Phi^i(\sigma) - x^i(\sigma)\mathbf{I}\,, \qquad \in \qquad \mathrm{SU}(N)\,, \tag{1.7}$$

which take values in SU(N), should be considered as pure worldvolume vector and scalar fields living on the N Dp-brane.

[2] I am thankful to Arkady Tseytlin for the discussion of this point

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Thus, to make the action (1.1) worldvolume reparametrization invariant we should rewrite it in a form where the center of mass coordinates x^i (1.5) are separated from the SU(N) scalar fields $\varphi^i(\sigma)$ (1.7).

Having written the N Dp-brane action in a reparametrization invariant form and having introduced the coordinates x^M of the target space, one may think of whether this construction can be generalized to be invariant under target space supersymmetry (by extending the target space to a superspace with Grassmann spinor coordinates) and under worldvolume (*abelian*) fermionic κ-symmetry which transforms the target superspace coordinates and "matter" fields on the brane.[3]

Note that in contrast to [9] we *a priori* assume the N Dp-brane action to be invariant under only one (abelian) κ-symmetry, since $N-1$ κ-symmetries of the initially separated N super-Dp-branes are regarded to have been gauge fixed.

The N Dp-brane action (1.1) has a rather complicated form so to simplify further analysis we shall restrict ourselves to a particular class of N coincident Dp-branes of codimension one, i.e. to the Dp-branes whose worldvolume has one dimension less than the dimension of the target space ($p+1=D-1$).

In this case we have only one U(N) adjoint scalar, hence all commutators of $\Phi(\sigma)$ in (1.4) and (1.1) vanish and the Wess-Zumino term does not have the contribution of couplings to higher-rank RR fields thus reducing to the non-abelian generalization of the standard WZ term of a single Dp-brane [11], as in the case of the N coincident space filling D-branes.

In section 2 we shall write a worldvolume reparametrization invariant action for a codimension one N Dp-brane and discuss its symmetry properties. In section 3 we shall simplify things even further by considering a system of N coincident D0-branes in a D=2 target space. We shall extend this system to an ND0-brane propagating in a type-IIA D=2 target superspace. The worldline reparametrization invariant action describing the dynamics of this system is shown to possess a target space supersymmetry, worldline κ-symmetry and a number of linearly and non-linearly realized rigid worldvolume supersymmetries. We find that the type-IIA $D=2$ superalgebra generating the target space supersymmetry transformations of the ND0-brane system acquires a non-trivial "central extension" due to a nonlinear contribution of the U(N) adjoint scalar fields. To the best of our knowledge this is the first example of a target-space supersymmetric and κ-invariant system of N coincident D-branes. In Conclusion we comment on the possibility of extending these results to higher-dimensional N Dp-branes.

2. The codimension one N Dp-branes

As we have discussed in the Introduction, in the case of codimension one Dp-branes

[3] Remember that in the case of the single superbranes the anticommutator of κ-symmetry always generates worldvolume diffeomorphisms.

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the action (1.1) reduces to

$$S = -T_p \int d^{p+1}\sigma \, \mathrm{Tr} e^{-\phi} \sqrt{-\det\left(P[G_{ab} + B_{ab}] + F_{ab}\right)} + T_p \int \mathrm{Tr} P \left[\sum C^{(n+1)} \, e^B\right] e^F \, , \tag{2.1}$$

where $C^{(n+1)}$ are (n+1)-form RR potentials with $n = p, p-2, p-4, \ldots$.

We now separate the transverse (center-of-mass) coordinate $x^{\perp}(\sigma)$ from the U(N) adjoint scalars and restore the worldvolume reparametrization invariance of (2.1) by introducing p+1 coordinates $x^a(\sigma)$ as in (1.5)-(1.7). The action (2.1) takes the form

$$\begin{aligned} S = T_p \int \mathrm{Tr} P \left[\sum C^{(n+1)} \, e^B\right] e^F - T_p \int d^{p+1}\sigma \, \mathrm{Tr} \, e^{-\phi} \, \times \qquad\qquad \qquad (2.2) \\ \times \sqrt{-\det(\partial_a x^M \partial_b x^N E_{MN} + F_{ab} + \partial_a x^M E_{M\perp} D_b\varphi + D_a\varphi E_{\perp N}\partial_b x^N + D_a\varphi G_{\perp\perp} D_b\varphi)}, \end{aligned}$$

where $\perp$ stands for the index of the target space direction, transverse to the brane worldvolume. Recall that in the case under consideration we have a single SU(N) adjoint scalar $\varphi(\sigma)$.

Looking at the form of the action (2.2) we see that though we have made it worldvolume reparametrization invariant the action has remained non-invariant under target space diffeomorphisms. One might expect this from the beginning, since, as we have already mentioned, the background fields and hence the action for N coincident Dp-branes depends on the U(N) adjoint scalars $\Phi^i = x^i \, \mathbf{I} + \varphi^i$ and not separately on x^M and φ^i. In particular, this means that in the flat background the N Dp-brane action does not possess target-space Lorentz invariance even in its center-of-mass part:

$$S = -T_p \int d^{p+1}\sigma \, \mathrm{Tr} \sqrt{-\det\left(\partial_a x^M \partial_b x^N \eta_{MN} + F_{ab} + D_a\varphi D_b\varphi + 2\partial_{(a} x^{\perp} D_{b)}\varphi\right)}, \tag{2.3}$$

where we have taken the background metric to be flat $\eta_{MN} = \mathrm{diag}(-, +, \cdots, +)$ and put all other background fields to zero.

If we forget about the space-time origin of the SU(N) adjoint fields $\varphi^i(\sigma)$ and (in accordance with our single N Dp-brane ideology) will consider them as non-abelian pure worldvolume scalars, which does not transform under target space Lorentz transformations but under an internal group SO(D-p-1), then the only term which spoils the target space Lorentz invariance of the N Dp-brane action is the last term in (2.3). Its presence implies that the dynamics of the center of mass of the system is directly affected by internal non-abelian fluctuations which is rather unusual.

If we drop this term we shall get a worldvolume diffeomorphism and target space Lorentz invariant brane system with worldvolume U(N) gauge fields $A_a(\xi)$ and an SU(N) adjoint scalar field φ propagating on the brane

$$S = -T_p \int d^{p+1}\sigma \, \mathrm{Tr} \sqrt{-\det\left(\partial_a x^M \partial_b x^N \eta_{MN} + F_{ab} + D_a\varphi D_b\varphi\right)}. \tag{2.4}$$

One may wonder whether these relativistic extended objects carrying a non-abelian matter on their worldvolumes may find a place for themselves in String Theory. We shall discuss an example of such systems in section 4.

Turning back to the N Dp-brane action (2.3), we can notice that, though being not Lorentz invariant, it is invariant under target space rigid translations

$$x^M \quad \to \quad x^M + a^M \,. \tag{2.5}$$

This allows us to assume that by introducing fermionic degrees of freedom the action (2.3) can be generalized to be invariant under rigid space-time supersymmetry transformations whose anticommutator closes on space-time translations. In addition, this model, being generalized in a proper way, can also possess a local fermionic κ-symmetry, whose anticommutator produces worldvolume diffeomorphisms.

In the next section, using a simple example of N D0-branes in D=2 space-time, we will demonstrate that such a generalization is indeed possible.

3. Supersymmetric and κ-symmetric ND0-brane model in type-IIA D=2 superspace

3.1 Bosonic ND0-brane system

In the case of the system of N coincident D0-branes in D=2 space-time parametrized by two coordinates x^M ($M = 0, 1$) the action (2.3) takes the form

$$S = -m \int d\tau \, \mathrm{Tr} \sqrt{- \left[\dot{x}^M \dot{x}^N \eta_{MN} + (\dot{\varphi})^2 + 2\dot{x}^1 \dot{\varphi}\right]}, \tag{3.1}$$

where m is the mass of the single D0-brane (the mass of the N D0-branes being, naturally, Nm), τ is the worldline time parameter and 'dot' denotes its derivative. Note that there is no place for the field strength of the gauge field $A(\tau)$ on the one dimensional worldline. Note also that since only $\dot{\varphi}$ enters the action, there is no ambiguity in choosing the U(N) trace. It is automatically symmetric.

By construction the action (3.1) is worldline reparametrization invariant, thus there must be the first class constraint which generates this symmetry. This constraint should be a generalization of (and reduce to) the mass shell condition for a massive relativistic particle of a mass Nm, when $\varphi = 0$

$$p_M p^M + (Nm)^2 = 0 \,, \tag{3.2}$$

where p_M is the particle momentum canonical conjugate to $x^M = (x^0, x^1)$.

Let us find the reparametrization constraint for the ND0-brane system.

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From the action (3.1) we derive the canonical momenta associated with x^0 and $\Phi = x^1\mathbf{I} + \varphi$, respectively,

$$p_0 = \frac{\delta S}{\delta \dot{x}^0} = -m\dot{x}^0 \mathrm{Tr}\left[\sqrt{-\left(\dot{x}^2 + \dot{\varphi}^2 + 2\dot{x}^1\dot{\varphi}\right)}\right]^{-1}, \tag{3.3}$$

$$p_\Phi = \frac{\delta S}{\delta \dot{\Phi}} = m\dot{\Phi}\left[\sqrt{(\dot{x}^0)^2 - \dot{\Phi}^2}\right]^{-1} \equiv m\dot{\Phi}\left[\sqrt{-\left(\dot{x}^2 + \dot{\varphi}^2 + 2\dot{x}^1\dot{\varphi}\right)}\right]^{-1} \quad (\text{no trace!}), \tag{3.4}$$

where $\dot{x}^2 \equiv \dot{x}^N\dot{x}^N\eta_{MN}$. The momentum conjugate to the spatial coordinate x^1 is the trace of p_Φ

$$p_1 = \frac{\delta S}{\delta \dot{x}^1} = m\mathrm{Tr}\left\{\dot{\Phi}\left[\sqrt{-\left(\dot{x}^2 + \dot{\varphi}^2 + 2\dot{x}^1\dot{\varphi}\right)}\right]^{-1}\right\}, \tag{3.5}$$

and the momentum associated with φ is

$$p_\varphi = p_\Phi - \frac{1}{N}p_1\,\mathbf{I}\,. \tag{3.6}$$

Upon some algebra we find the following constraint on the momenta

$$-(p_0)^2 + \left(Tr\sqrt{p_\Phi^2 + m^2\,\mathbf{I}}\right)^2 = 0\,. \tag{3.7}$$

As one can immediately check, in view of (3.6), the constraint (3.7) reduces to (3.2) when the adjoint field $\varphi(\tau)$ is zero.

For further supersymmetrization it is convenient to rewrite the action (3.1) in the first order form

$$S = \int d\tau \mathrm{Tr}\left\{\frac{1}{N}p_M\dot{x}^M + p_\varphi\dot{\varphi} - \frac{e(\tau)}{2N}\left[-(p_0)^2 + \left(Tr\sqrt{p_\Phi^2 + m^2\,\mathbf{I}}\right)^2\right]\right\}, \tag{3.8}$$

where $e(\tau)$ is the Lagrange multiplier ensuring the constraint (3.7).

We are now in a position to add to the system fermionic modes and lift it to the type-IIA $D = 2$ superspace. To this end let us first remind the structure and the symmetries of the action for a single super-D0-brane, i.e. a massive type-IIA superparticle.

3.2 The massive type-IIA superparticle

The description of various aspects of the dynamics, group-theoretical and geometrical properties of the massive superparticle in type-IIA $D = 2$ superspace the reader may find e.g. in [12, 13]. Similar to the $D = 10$ case the type-IIA $D = 2$ superspace is parametrized, in addition to the bosonic coordinates x^M, by a two-component (real) Majorana spinor, or two Majorana-Weyl coordinates of different chirality

$$\theta^\alpha = (\theta^1, \theta^2)\,. \tag{3.9}$$

In the first order formalism the type-IIA $D = 2$ superparticle action has the following form

$$S = \int d\tau \left\{ p_M(\dot{x}^M + i\theta\gamma^M\dot{\theta}) - \frac{e(\tau)}{2}(p_M p^M + m^2) + im\,\theta\gamma^2\dot{\theta} \right\}, \tag{3.10}$$

where the last term in (3.10) is the Wess-Zumino or the Chern-Simons term, and $\gamma^M_{\alpha\beta}$ (M=0,1) and $(\gamma^2)_{\alpha\beta}$ are $D = 2$ Dirac matrices in the Majorana representation:

$$\gamma^0_{\alpha\beta} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad \gamma^1_{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \gamma^2_{\alpha\beta} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{3.11}$$

The charge conjugation matrix C for raising and lowering spinor indices is

$$C_{\alpha\beta} = C^{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{3.12}$$

The action (3.10) is invariant under the $D = 2$ Lorentz rotations, Poincare translations, global supersymmetry

$$\delta_\epsilon\theta^\alpha = \epsilon^\alpha, \qquad \delta_\epsilon x^M = -i\epsilon\gamma^M\theta, \qquad \delta_\epsilon P_M = 0 = \delta_\epsilon e \tag{3.13}$$

and local fermionic κ-symmetry

$$\delta_\kappa\theta = (p_M\gamma^M + m\gamma^2)\kappa(\tau), \quad \delta_\kappa x^M = i\delta\theta\gamma^M\theta, \quad \delta_\kappa e = 4i\kappa^\alpha\dot{\theta}_\alpha, \quad \delta_\kappa p_M = 0, \tag{3.14}$$

where $\kappa^\alpha(\tau)$ is a Grassmann-odd Majorana spinor parameter of the κ-symmetry.

Note that due to the Dirac matrix algebra the square of $(p_M\gamma^M + m\gamma^2)$ in the κ-variation of θ is proportional to the mass shell condition

$$(p_M\gamma^M + m\gamma^2)(p_N\gamma^N + m\gamma^2) = (p_M p^M + m^2)\,C. \tag{3.15}$$

This means that only half of $\kappa^\alpha(\tau)$ effectively contributes to the κ-variations (3.14), and that κ^α can eliminate only one of the components of θ^α by imposing, say a covariant gauge $\theta^1 = 0$.

In this gauge the action takes the form

$$S = \int d\tau \left\{ -p^0(\dot{x}^0 + i\theta^2\dot{\theta}^2) + p_1\dot{x}^1 - \frac{e(\tau)}{2}(p_M p^M + m^2) + im\,\theta^2\dot{\theta}^2 \right\}. \tag{3.16}$$

Finally rescaling θ^2 as

$$\chi = \sqrt{(p^0 - m)}\,\theta^2 \tag{3.17}$$

we arrive at the action with the free fermion kinetic term

$$S = \int d\tau \left\{ -p^0\dot{x}^0 + p_1\dot{x}^1 - \frac{e(\tau)}{2}(p_M p^M + m^2) - i\chi\dot{\chi} \right\}. \tag{3.18}$$

We have now all ingredients to supersymmetrize the action for the N coincident D0-branes by analogy with the single superparticle action.

3.3 The supersymmetric and κ-invariant ND0-brane action

We assume that when the κ-symmetry is gauge fixed the supersymmetric ND0-brane action has the form analogous to (3.18), but with the bosonic part as in eq. (3.8) and the abelian fermion χ being extended to a U(N) adjoint fermion

$$\Psi(\tau) = \chi(\tau)\,\mathbf{I} + \psi(\tau)\,, \tag{3.19}$$

where $\psi(\tau)$ is a traceless fermionic matrix.

We thus write the κ-gauge fixed super-ND0-brane action in the following form

$$S = \int d\tau \mathrm{Tr}\left\{\frac{1}{N}p_M\dot{x}^M + p_\varphi\dot{\varphi} - \frac{e(\tau)}{2N}\left[-(p_0)^2 + \left(\mathrm{Tr}\sqrt{p_\Phi^2 + m^2\,\mathbf{I}}\right)^2\right] - i\Psi\dot{\Psi}\right\},$$

or

$$S = \int d\tau \mathrm{Tr}\left\{\frac{1}{N}p_M\dot{x}^M + p_\varphi\dot{\varphi} - \frac{e(\tau)}{2N}\left[-(p_0)^2 + \left(\mathrm{Tr}\sqrt{p_\Phi^2 + m^2\,\mathbf{I}}\right)^2\right] - i\chi\dot{\chi} - i\psi\dot{\psi}\right\}. \tag{3.20}$$

Note that the action (3.20) does not contain non-diagonal fermionic terms of the form $\chi\dot{\psi}$ which vanish because ψ is traceless. If we in addition impose the static gauge $x^0 = \tau$ the action reduces to

$$S = \int d\tau \mathrm{Tr}\left\{p_\Phi\dot{\Phi} - \sqrt{p_\Phi^2 + m^2\,\mathbf{I}} - i\chi\dot{\chi} - i\psi\dot{\psi}\right\}, \tag{3.21}$$

where $H = Tr\sqrt{p_\Phi^2 + m^2\,\mathbf{I}}$ is the hamiltonian of the gauge fixed system.

We would now like to restore the manifest global supersymmetry (3.13) of the action (3.20) by adding θ^1 and interpreting χ as a θ^2-component of the Grassmann spinor coordinate θ^α (3.9). Upon having appropriately rescaled χ and by analogy with (3.10) we can write the following action

$$\begin{aligned} S = \int d\tau \mathrm{Tr}\Bigg\{&\frac{1}{N}p_M(\dot{x}^M + i\theta\gamma^M\dot{\theta}) + p_\varphi\dot{\varphi} - \frac{e(\tau)}{2N}\left[-(p_0)^2 + \left(\mathrm{Tr}\sqrt{p_\Phi^2 + m^2\,\mathbf{I}}\right)^2\right] + \\ &+ im\,\theta\gamma^2\dot{\theta} - i\psi\dot{\psi}\Bigg\}. \end{aligned} \tag{3.22}$$

The action (3.22) is invariant under the global supersymmetry transformations (3.13) but it is not yet κ-symmetric. The reason is that the mass shell condition (3.7) of the ND0-brane system differs from the mass shell condition (3.15) of the single superparticle. Let us rewrite (3.7) in the following form

$$p_Mp^M + \left[\left(\mathrm{Tr}\sqrt{p_\Phi^2 + m^2\,\mathbf{I}}\right)^2 - (p_1)^2\right] = 0\,. \tag{3.23}$$

Comparing (3.23) with (3.15) we see that the term of (3.23) in the square brackets plays the role of an "effective mass" of the ND0-brane system

$$M(p_\varphi, p_1) \equiv \left[\left(\mathrm{Tr}\sqrt{p_\Phi^2 + m^2\,\mathbf{I}} \right)^2 - (p_1)^2 \right]^{1/2} . \tag{3.24}$$

Thus to restore κ-symmetry we should replace m with $\frac{M}{N}$ in the κ-symmetry variation rules (3.14) and in the term $im\theta\gamma^M\dot{\theta}$ of the action (3.22).

The final form of the supersymmetric and κ-invariant ND0-brane action is

$$S = \int d\tau \mathrm{Tr}\Big\{ \frac{1}{N} p_M(\dot{x}^M + i\theta\gamma^M\dot{\theta}) + p_\varphi\dot{\varphi} - \frac{e(\tau)}{2N}\left[p_M p^M + M^2(p_\varphi, p_1) \right] + \\ + i\frac{M(p_\varphi, p_1)}{N}\theta\gamma^2\dot{\theta} - i\psi\dot{\psi} \Big\} . \tag{3.25}$$

The action (3.25) is invariant under the following κ-symmetry variations

$$\delta_\kappa\theta = \left(p_M\gamma^M + M(p_\varphi, p_1)\gamma^2 \right)\,\kappa(\tau), \quad \delta_\kappa x^M = i\delta\theta\gamma^M\theta \, + \, i\delta\theta\gamma^2\theta\,\frac{\delta M(p_\varphi, p_1)}{\delta p_1}\,\delta_1^M , \tag{3.26}$$

$$\delta_\kappa e = 4i\kappa^\alpha\dot{\theta}_\alpha, \tag{3.27}$$

$$\delta_\kappa\varphi = i\delta\theta\gamma^2\theta\,\left[\frac{\delta M(p_\varphi, p_1)}{N\delta p_\varphi} \right]_{\mathrm{traceless}} , \tag{3.28}$$

where $\frac{\delta M(p_\varphi, p_1)}{\delta p_1}$ and $\frac{\delta M(p_\varphi, p_1)}{\delta p_\varphi}$ are functions obtained by varying $M(p_\varphi, p_1)$ with respect to p_1 and p_φ, respectively. The ψ-fields are inert under the κ-symmetry.

We should note that since $M(p_\varphi, p_1)$, being a function of p_φ and p_1, is not constant the global supersymmetry transformation of the spatial coordinate x^1 also gets modified (as under κ-symmetry (3.26)) and takes the form

$$\delta_\epsilon x^1 = -i\epsilon\gamma^1\theta - i\epsilon\gamma^2\theta\,\frac{\delta M(p_\varphi, p_1)}{\delta p_1} . \tag{3.29}$$

This modified supersymmetry variation is a price for the model to be non-invariant under $D = 2$ Lorentz transformations. The Lorentz invariance of the first order formulation is broken by an explicit dependence of M on p_1. In the next section we shall consider a Lorentz-covariant counterpart of this system with the standard supersymmetry transformations of the type-IIA superspace.

Because of the functional dependence of $M(p_\varphi, p_1)$ also the SU(N) adjoint scalar φ transforms nontrivially under the space-time supersymmetry, namely,

$$\delta_\epsilon\varphi = -i\epsilon\gamma^2\theta\,\left[\frac{\delta M(p_\varphi, p_1)}{N\delta p_\varphi} \right]_{\mathrm{traceless}} . \tag{3.30}$$

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The modified global supersymmetry transformations (3.29), (3.30) of x^1 and φ close on generalized global bosonic 'translations' of x^1 and φ under which the action (3.25) is also invariant:

$$\delta x^1 = a^1 + b\,\frac{\delta M(p_\varphi, p_1)}{\delta p_1}, \quad \delta\varphi = A + b\,\left[\frac{\delta M(p_\varphi, p_1)}{N\delta p_\varphi}\right]_{\text{traceless}}, \tag{3.31}$$

where a^1 and A are parameters of the standard 'non-abelian' translations and b is the parameter of the additional global bosonic symmetry.

It is instructive to present the form of the supersymmetry algebra of the transformations (3.29)-(3.31) generated by the Poisson brackets of the Noether supercharges derived from the action (3.25)

$$\{Q_\alpha, Q_\beta\} = 2ip_M\gamma^M_{\alpha\beta} + 2iM(p_\varphi, p_1)\,\gamma^2_{\alpha\beta}\,, \tag{3.32}$$

where

$$Q_\alpha = \pi_\alpha + ip_M(\gamma^M\theta)_\alpha + iM(p_\varphi, p_1)\,(\gamma^2\theta)_\alpha\,, \tag{3.33}$$

and π_α is the momentum conjugate to θ^α.

We observe that the superalgebra (3.32) has the "central charge" term proportional to the effective mass $M(p_\varphi, p_1)$ which arises because the U(N) adjoint scalars nontrivially transform under supersymmetry. Such a "central extension" of the superalgebra is akin to that of the superalgebras associated with the single superbranes having the Wess-Zumino terms in their actions [14] and gauge fields on their worldvolumes which vary under target-space supersymmetry transformations [15].

When the adjoint scalar field $\varphi(\tau)$ is switched off, $p_\varphi = 0$, $M(0, p_1) = Nm$ and the superalgebra (3.32) reduces to the standard type-IIA superalgebra with the central charge Nm associated with the massive superparticle of subsection 3.2.

Finally the ND0-brane action (3.25) is invariant under the following rigid worldvolume linear supersymmetry transformations of φ and ψ with a U(N) adjoint fermionic parameter α

$$\delta\psi = [\alpha p_\varphi]_{\text{traceless}}\,, \qquad \delta\varphi = -2i[\psi\alpha]_{\text{traceless}}\,. \tag{3.34}$$

If we assume that in eq. (3.25) the trace is symmetrized then the action is also invariant under non-linearly realized non-abelian worldline supersymmetry transformations of ψ

$$\delta\psi = \left[\beta(\mathbf{I} + if(\tau)\psi\dot{\psi})\right]_{\text{traceless}}\,, \tag{3.35}$$

where β is a constant SU(N) adjoint fermionic parameter and $f(\tau)$ is an arbitrary adjoint scalar function. Under (3.35) the adjoint fermion transforms as a Goldstone field, thus the non-abelian supersymmetry (3.35) is spontaneously broken. These non-abelian supersymmetries may be regarded as a relic of the space-time supersymmetry of $(N-1)$ D0-branes of the ND0-brane system whose κ-symmetries have been gauge fixed.

We also notice that if in (3.25) we take the symmetrized trace the action cannot acquire higher order corrections in ψ and $\dot\psi$ since they vanish identically.

The abundance of the worldvolume rigid supersymmetries of the ND0-brane action in $D = 2$ is connected with its free fermion structure. For N Dp-branes with $p \geq 0$ and $D > 2$ higher order fermionic terms should appear in the action and worldvolume supersymmetries should be related to the space-time supersymmetry.

To conclude this section let us note that the form of the action (3.25) agrees with a supersymmetric action for N D0-branes in Matrix Theory [2] reduced to $D = 2$.

4. The Lorentz invariant super-ND0-brane

We now present a Lorentz invariant counterpart of the ND0-brane model considered in the previous section.

If in (3.1) we skip the term $\dot x \dot\varphi$ the model becomes $D = 2$ Lorentz invariant. The reparametrization constraint takes the Lorentz-covariant form

$$p_M p^M + \left(\mathrm{Tr}\sqrt{p_\varphi^2 + m^2\,\mathbf{I}}\right)^2 = 0\,, \tag{4.1}$$

and so does the effective mass M

$$M(p_\varphi) = \mathrm{Tr}\sqrt{p_\varphi^2 + m^2\,\mathbf{I}}\,. \tag{4.2}$$

The supersymmetric and κ invariant action for the Lorentz invariant ND0-brane is easily deduced from (3.25)

$$S = \int d\tau \mathrm{Tr}\Big\{\frac{1}{N} p_M(\dot x^M + i\theta\gamma^M\dot\theta) + p_\varphi\dot\varphi - \frac{e(\tau)}{2N}\left(p_M p^M + M^2(p_\varphi)\right) + \\ + i\frac{M(p_\varphi)}{N}\,\theta\gamma^2\dot\theta - i\psi\dot\psi\Big\}\,. \tag{4.3}$$

The variation properties of the fields with respect to the symmetries of the action (4.3) are the same as written in equations (3.13), (3.26)–(3.35), except that now the spatial coordinate x^1 transforms in the standard way under the κ-symmetry, $D = 2$ supersymmetry and translations. This is because now $M(p_\varphi)$ does not depend on p_1. The model is Lorentz invariant and the type IIA supersymmetry algebra is standard when acting on x^M but it is extended by the "central" charge $M(p_\varphi)$ (as in (3.32)) due to the presence of the worldvolume adjoint scalar $\varphi(\tau)$, which transforms non-trivially under the target space supersymmetry and κ-symmetry.

5. Conclusion

In this paper we have examined a particular class of N coincident Dp-branes of codimension one. We have seen that for these branes the structure of the worldvolume action significantly simplifies. We have then made the action invariant under worldvolume diffeomorphisms. Though this has not restored target space Lorentz symmetry, the action became invariant under the space-time translations. The invariance of the action under worldvolume diffeomorphisms and target space translations has allowed us to assume that the system of N Dp-branes of codimension one can be generalized to include fermions in a target space supersymmetric and worldvolume κ-symmetric fashion.

We have demonstrated that such a generalization is indeed possible by constructing the supersymmetric and κ-invariant action which describes N coincident D0-branes in type-IIA $D = 2$ superspace.

We have noticed that by dropping some terms in the N Dp-brane action and assuming the SU(n) adjoint scalars to be pure worldvolume fields we can make the action Lorentz invariant, and presented the supersymmetric and κ-invariant action for a Lorentz covariant system of N D0-branes.

A natural generalization of the results of this paper would be to try to construct corresponding supersymmetric and κ-invariant actions for ND-brane systems of higher dimension and codimension. The prescription of how to act is prompted by the example of the ND0-brane system. One should write down the action for bosonic N Dp-branes in a worldvolume diffeomorphism invariant form, find dynamical constraints which generate the worldvolume diffeomorphisms and then try to modify in an appropriate way the action and the projector matrix in the κ-symmetry transformations of the corresponding single super-Dp-brane.

An important and interesting problem to think about is Lorentz (non)invariance of the systems of N coincident Dp-branes. The formalism of Lorentz harmonics might be useful in studying this problem [16].

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